Appendix One: Additional Background Material and Exercises for Students

For each chapter except the Introduction and Chapter Nine I've provided additional background material followed by several questions. In most cases the material provided here or in the main section of the book is sufficient to answer the questions, but in a few cases students will need to consult outside sources. Spoiler alert: if you are using the version downloaded from the Cambridge web site, answers are listed immediately after each question. Instructors using this material to lead in-class discussions (the Socratic teaching method) may find this format helpful.

In this version, on the author's web site, the answers have been grouped at the end of the Appendix. Instructors wishing to use the material as problem sets to be handed out and wrestled with by students will find this format more useful.

Chapter Two

What is a Fault? A fault is a planar fracture in the Earth's crust that develops along a zone of weakness and forms a boundary between Earth's plates. Since the plates move in different directions and at different speeds, there is relative motion between them, and that motion is accommodated on the bounding faults. Most earthquakes occur on faults, so geologists spend a lot of time studying them. Since we live in a three dimensional world, it is convenient to define three types of faults, based on the sense of relative motion (sideways, up, or down). If we define a moving block or plate and a stable reference block or plate, a strike-slip fault is one in which the moving block moves sideways relative to the reference; a thrust fault is one in which the moving block is pushed up relative to the reference, and a normal fault is one in which the moving block slides down relative to the reference. In this book we are mainly concerned with thrust faults and strike slip faults (Figure A 2.1). Note the difference in the slopes of the fault planes: the thrust fault slopes downward (geologists call this the dip angle), while the strike-slip fault is vertical. Thrust faults are the main ones in subduction zones, causing the biggest earthquakes and most tsunamis (e.g., the Fukushima disaster described in Chapter Four). The San Andreas fault in California (Chapter Two) and the North Anatolian fault in Turkey (Chapter Five) are examples of strike-slip faults.
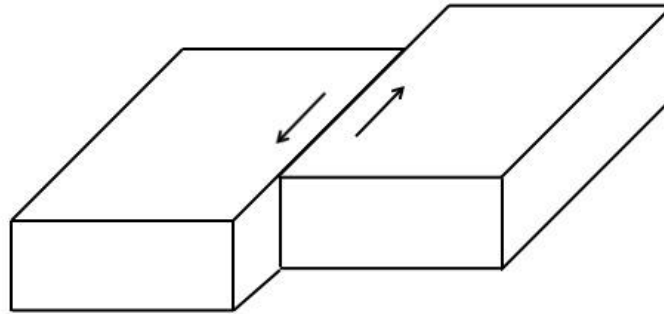
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Thrust Fault
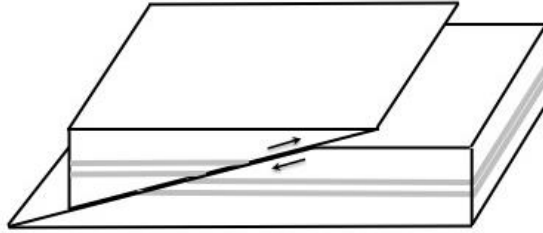
Strike Slip Fault

Figure A 2.1. Sense of relative motion for thrust faults (top) and strike slip faults (bottom).

Stick-Slip, Strain Accumulation, Elastic Rebound and Earthquakes. In the elastic rebound model, described in Chapter Two and first articulated by geologist H. F. Reid, earthquakes occur because of relative motion between two plates or crustal blocks (essentially small plates) separated by a fault, combined with friction on that fault. Without friction, the blocks or plates could easily slide by each other, and the dangerous build-up of elastic strain would not occur. But friction on faults is the norm, at least in the upper 15-30 km of Earth's crust. Below this depth, the Earth is usually warm enough that rocks behave in a ductile manner (they flow like toothpaste) and the plates can more or less slide by each other. Because the upper part of the fault is locked (stuck), rocks near the fault will bend, and elastic strain builds up in that volume of rock.

The relative displacements that occur during the pre-earthquake strain accumulation phase (the "stick" part of the stick-slip cycle described in Box 2.2) are quite small, but they can be measured with modern geodetic techniques such as high precision GPS. Figure A 2.2 shows an example from the San Andreas fault in California.
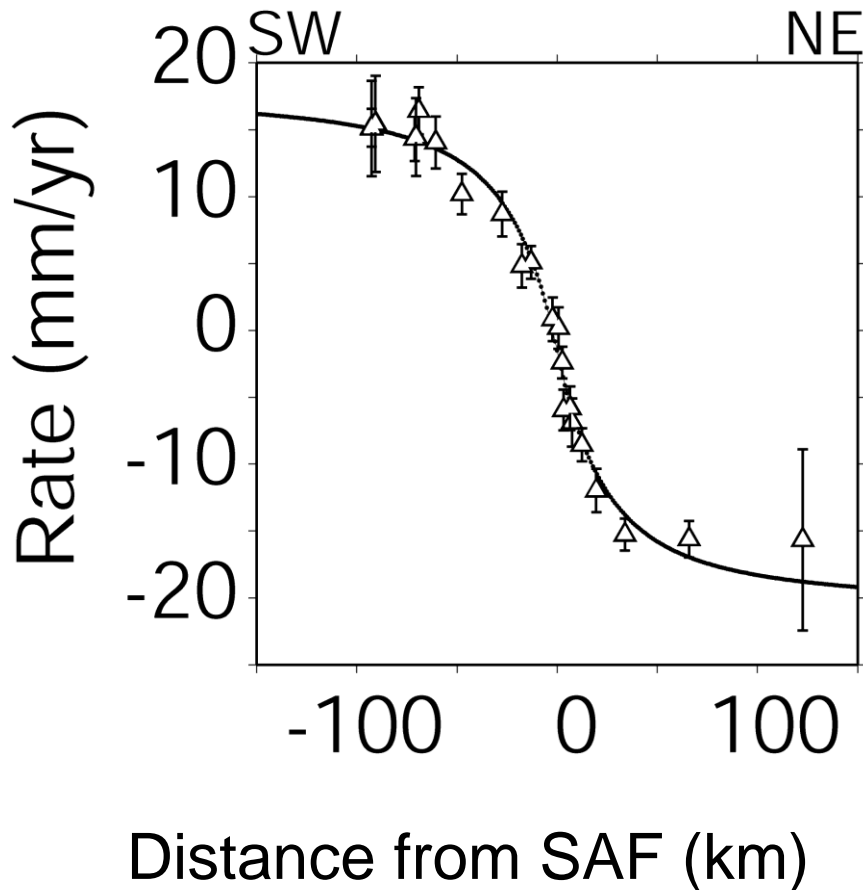
Figure A 2.2 GPS displacement data (triangles, with vertical lines showing measurement uncertainty) across the San Andreas Fault (SAF) in central California, showing the effect of elastic strain accumulation. The displacement data were measured between 1994 and 2003, on stations that defined a profile perpendicular to the fault. The "S" shaped curve shows a model fit to the GPS data, and indicates strain accumulation in the Earth's upper crust (the "stick" part of the stick-slip earthquake cycle; compare to the figure in Box 2.3, middle panel). Modified from *Schmalzle et al.* (2005).

Question 2-1. A small stream bed that flows across the San Andreas fault is offset about 150 meters. Trenching in a wetland associated with the stream suggests that the earliest stream deposits were laid down 5000 years ago. What is the long-term slip rate on the fault?

Question 2-2. Refer to Figure A 2.2: what is the rate of motion of a point 100 km northeast of the San Andreas fault relative to a point 100 km southwest of the fault?

Earthquake Size, Rupture Area and Fault Type. Earthquake size is usually described by a magnitude scale (M), where each unit difference represents about a factor of 30 difference in energy release. The earthquake magnitude is related to the area of fault rupture, and the amount of slip during the rupture. More area, or more slip, means bigger

earthquakes. The rupture area is that part of the fault that actually moves in the earthquake. Subduction zones produce Earth's largest earthquakes, up to M ~ 8-9. Typically, strike slip earthquakes (where the motion across a vertical fault is mostly horizontal) reach smaller maximum sizes, M ~7-8, although there can be exceptions. It's interesting to see what causes the size difference. The area of fault rupture turns out to be a key factor. For a typical strike slip fault, which tends to be oriented vertically, the surface area of fault rupture is just the depth of faulting times the length of the fault rupture on the surface. The depth is typically 10-15 km; below that, rocks are so hot that they can flow like toothpaste (at least on long time scales), and hence do not accumulate elastic strain energy.

Question 2-3. Compare the relative energy release of earthquakes occurring on a vertical strike slip fault versus a thrust fault dipping at 10 degrees from horizontal, assuming each is 100 km long and the maximum depth of seismic rupture in 15 km.

Another factor that makes subduction earthquakes bigger (because it also influences fault area) has to do with temperature. Hotter rocks tend to flow like toothpaste, and don't accumulate seismic strain. The temperature cut-off for earthquakes in subduction zones is actually deeper than 15 km, because the process of subduction cools the upper few hundred km of the Earth. The subducted crust and lithosphere are cooler than their surroundings, so those surroundings (the deeper parts of the over-riding plate) are cooled by the subducted crust and lithosphere, much like an ice cube cools off of a drink. The net effect is that the typical depth cut-off for earthquakes is closer to 30 km than 15 km, doubling our hypothetical thrust fault area, to about 17,200 km^2. If we add this effect to the answer to Question 2-3, our hypothetical thrust fault produces an earthquake that is a factor of eleven bigger than our hypothetical strike-slip fault.

The last factor affecting fault area and earthquake size has to do with fault length: the largest subduction zone earthquakes rupture much longer fault segments compared to strike slip faults, for reasons that are not well understood. In the 2004 Sumatra earthquake, a subduction zone event, nearly 1,200 km of the plate boundary ruptured over a period of several minutes. In contrast, the great San Francisco earthquake of 1906, the largest known earthquake on California's San Andreas fault, a strike-slip fault, only ruptured ~400-500 km of the plate boundary.   Of course, this difference may reflect our short observational record – perhaps we have not yet observed the longest strike slip fault rupture.

Chapter Three

Earthquake Recurrence Intervals and Aliasing. Determining earthquake recurrence intervals is a critical part of hazard assessment. In this section I'll consider how the concept of aliasing affects the measurement and interpretation of these intervals. Figure A 3.1 illustrates an example of aliasing, and introduces the concept of Nyquist frequency, the minimum sampling interval necessary to adequately describe a signal.
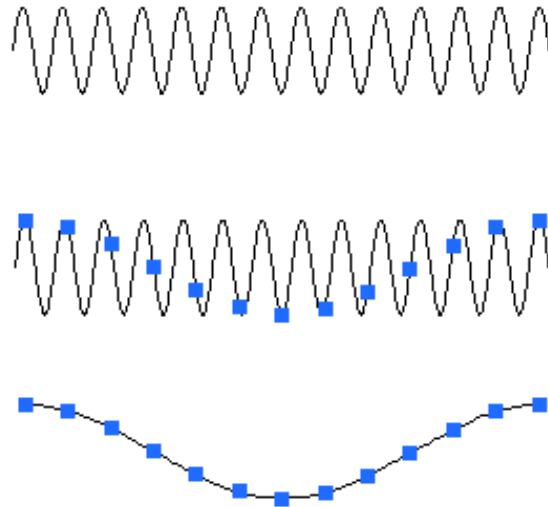
Figure A 3.1 Aliasing. The top graph is a representation of the true signal (the vertical axis might be voltage or current, while the horizontal axis is time). The sinusoidal signal appears somewhat choppy because it has been slightly under-sampled (20 samples per cycle) but the original smooth sinusoidal shape is still obvious. The middle graph shows an example of highly aliased sampling (small squares, less than one sample per cycle, total 13 samples). The bottom graph shows the signal we would infer if we only had those 13 samples. If one wished to determine the period of the signal (the time between the peaks), the aliased signal would give a result that is far too long. Adequate sampling would require that, at a minimum, we sample every peak and valley (two samples per cycle). This type of sampling (the minimum necessary to determine the true period) is called the Nyquist frequency, after the electrical engineer who first described it.

Let's see how the Nyquist criteria, and the more general concept of aliasing and appropriate sampling, might apply to the idea of earthquake recurrence intervals. We'll first assume that earthquakes occur in a regular, periodic way. By comparing an earthquake record to a sinusoidal signal like the one shown in Figure A.3.1, we'll assume that an earthquake is a "peak", while a period of no earthquakes is a "valley". We mainly need to assure ourselves that the trenching or other stratigraphic data that is available samples all the big earthquakes in a given location and period of time, making sure we did not miss a major event in between (which would indicate that the earthquakes are more frequent). One way to do this is to use several trenches – if they give the same answer, we are probably on to something. We also need to make sure that the sampling period is sufficiently long to be representative – in other words, is it truly periodic? Three complete cycles is generally considered a minimum.

Question 3-1. We wish to test the hypothesis that a major earthquake hits a given location every 200 years. How well does the geologic record need to be sampled, and how far back in time would we need to go?

Now lets assume that earthquakes do not occur in a simple periodic way. What if, as some geologists suspect, earthquakes tend to occur in clusters? How would this affect our sampling criteria?

Question 3-2. We hypothesize that major earthquakes strike a given location every 200 years three times in a row, after which there are no earthquakes for 600 years, then three more, etc. Assuming this behavior is characteristic, what is the period of the overall sequence, and how long would your record have to be to say with reasonable confidence that you have adequately sampled the phenomenon?

Aliasing, and assignment of risk based on a too-short earthquake record, likely played a role in the disaster at Fukushima, discussed in Chapter Four. The challenges inherent in a short historical record are discussed in Chapter Five, in the section on the Cascadia subduction zone ("Sleepless in Seattle").

Chapter Four

Cesium (chemical symbol Cs) is an alkali metal, similar to sodium (Na) and potassium (K) in its chemical properties. It has an atomic number of 55, meaning it has 55 protons in its nucleus, and a variable number of neutrons, depending on which isotope is under discussion. Isotopes refer to the atomic weight of a particular atom (the sum of protons and neutrons): ^{137}Cs has 82 neutrons and 55 protons. Since $82 + 55 = 137$, it has an atomic weight of 137.

When a uranium atom (atomic number 92, atomic weight 235) splits apart (fissions) in a nuclear reactor (usually when it is hit with an extra neutron), ^{137}Cs is one of the common products. It is unstable, and decays with a half-life of about 30 years (Figure A.4.1). It is also toxic, but is absorbed by most plants and animals because of its chemical similarity to potassium, an element critical for life in most organisms (including people). It's important to be able to measure the mount of ^{137}Cs in the environment after a nuclear accident.

Question 4-1. Assume an atom of ^{235}U fissions in a nuclear reactor, by absorbing a neutron (temporarily changing it to ^{236}U), and then splitting into two lower weight fragments, one of which is ^{137}Cs. What is the atomic weight of the other fragment, assuming that 3 neutrons are given off in the nuclear reaction?
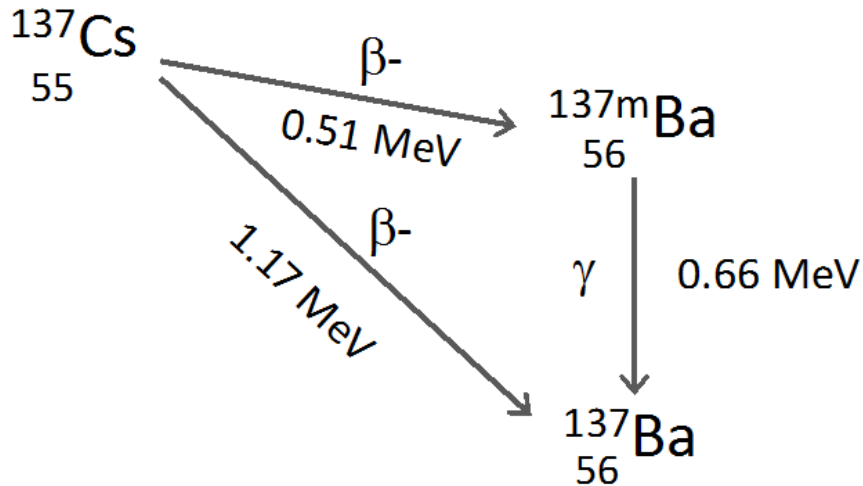
Figure A 4.1 ^{137}Cs decays via emission of a beta particle (high speed electron, symbol β-) to either a stable form of barium (^{137}Ba) or an unstable form, known as an isomer (^{137m}Ba, where the *m* stands for isomer). In the latter case, which occurs about 95% of the time, the unstable form decays almost immediately to the stable form by emitting a Gamma ray (symbol γ). The energy of each emission, shown here in units of millions of electron-volts (MeV) is always the same, 0.66 MeV for the Gamma radiation, 0.51 and 1.17 MeV for the Beta particles.

Question 4-2. Your first job after graduation is to design a cheap, portable instrument that will measure the amount of ^{137}Cs in the environment after a nuclear accident, using low cost commercial off the shelf components (sometimes called COTS components). You find that there are low cost devices for counting Beta particles with energies between 1.1 and 1.2 MeV, and equally low cost counters for Gamma rays with energies between 0.6 and 0.7 MeV. If all other things are equal, which would you choose as the basis for your device? Why?

Chapter Five

Question 5-1. Based on the material presented in this chapter, come up with two different ways of estimating the year when a major earthquake will hit Istanbul. How would you calculate the uncertainty for each estimate?

Chapter Six

Question 6-1: Lead in Cities. Figure 6.2, reproduced below in color as Figure A 6.1, shows a pattern of lead concentration that is quite common in the soil of many cities. In Chapter 6, this was mostly attributed to leaded gasoline. List other possible sources of lead contamination in urban areas. How would you go about deciding which of these sources is most important?
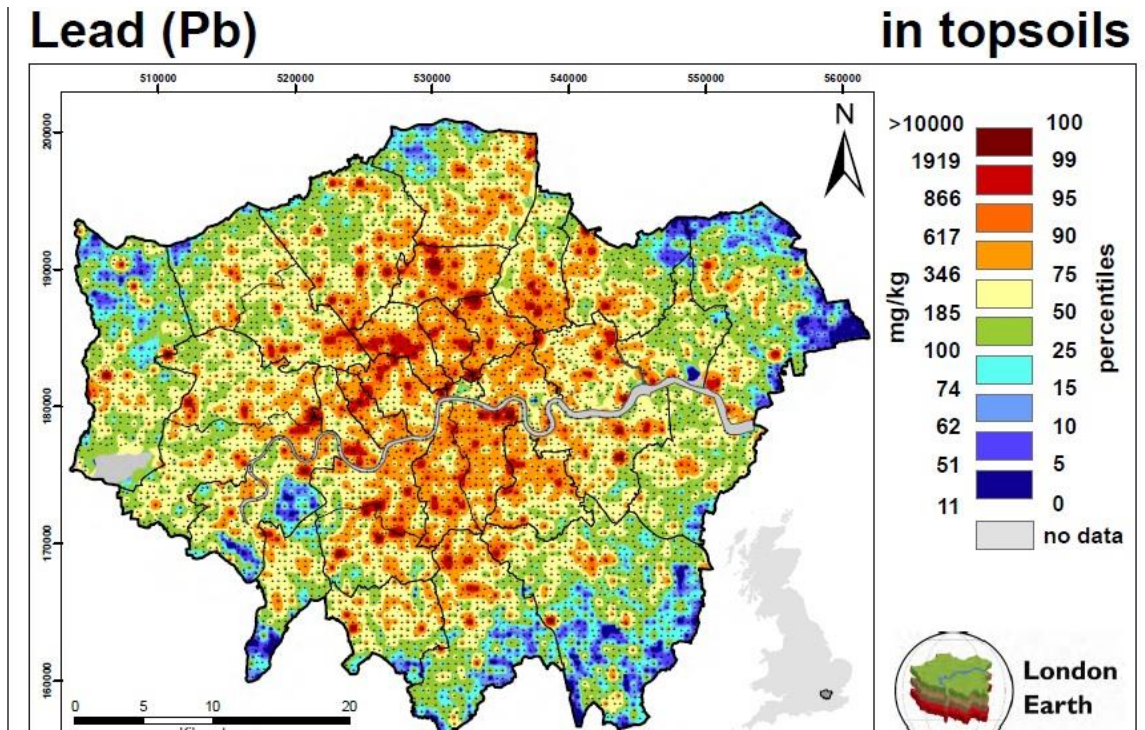
Figure A 6.1. Lead in topsoil in London, England, measured by the British Geological Survey in 2008-2009. The scale bar on the right is in units of milligrams (mg) of lead per kilogram (kg) of soil (mg/kg). Levels above 100 mg/kg are generally considered unsafe, although the Dutch government sets a standard of 40mg/kg. Note that most of central London exceeds even the higher value. Most major urban centers show similar levels of lead contamination in their topsoil. Reproduced by permission of the British Geological Survey © NERC. All rights reserved. CP14/021.

Boom and Bust cycles, the price of oil, and population growth – the magic of differential equations. The Lotka - Volterra equations are a pair of differential equations describing the behavior of two variables that depend on each other. They are often called the predator-prey equations because under certain idealized conditions they describe the time-dependent population of two species such as foxes (predators) and rabbits (prey):

$$\frac{dx}{dt} = a_1 x - a_2 xy \qquad\qquad \text{A6.1a}$$

$$\frac{dy}{dt} = -b_1 y + b_2 xy \qquad\qquad \text{A6.1b}$$

where x is the population of prey at any time t, y is the population of predators at any time, and a_1, a_2, b_1 and b_2 are constants that describe how the two species interact. Solutions to this equation usually have the form of two oscillating curves, with the second (y as a function of time) slightly lagging the first (x as a function of time).

A classic example is based on a long time series of data on lynx and snowshoe hare populations in North America collected by the Hudson Bay Company in the 1800's and early 1900's (Figure A 6.2). The company provided a market and clearing house for furs collected by trappers.
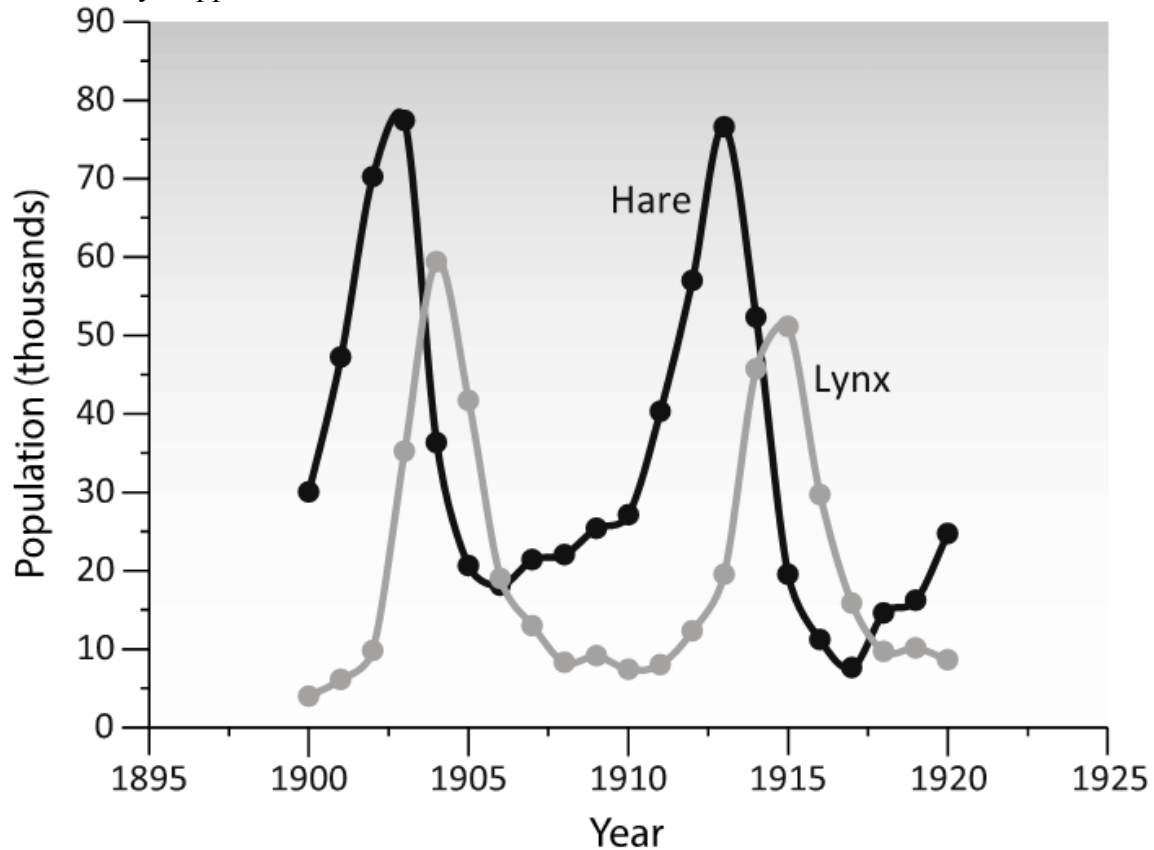

Figure A 6.2. Population trends for snowshoe hares and lynxes in the Canadian boreal forest, based on data collected by the Hudson Bay Company.

The data set is actually the number of pelts purchased by the company, but this is a reasonable proxy for overall population. The two time series show a decadal cycle for each population, with peaks in the lynx population lagging peaks in the hare population by one to two years. It is usually explained as follows. Initially, the population of both hares (prey) and lynxes (predators) is low. Since predation levels are low, the hare population grows rapidly. Eventually, the lynxes start to benefit from the increased food supply, and their numbers also grow. But with increased predation, the number of hares falls. Within a year or two, this loss of food supply starts to affect the lynxes, their population also falls, and the cycle starts again. By choosing appropriate values of a_1, a_2, b_1 and b_2, in equations A6.1a and b, we could generate curves (models) that closely match the hare and lynx data in Figure A 6.2.

As with any model, there are certain simplifying assumptions that need to be kept in mind. For example, the model assumes that the prey has no other predators, and the predators have no other food source. Also, the prey must be sufficiently diverse or clever such that they cannot be wiped out by the predators, otherwise both populations would

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soon go to zero. In reality, population dynamics are more complex. While lynxes mainly depend on hares for food, hares have other predators. The hare population is also affected by disease (which can also cause cyclical effects) and food availability, which responds to climate trends. Nevertheless, the model is successful at explaining some of the first order controls on the hare and lynx populations.

Question 6-2. With reference to equation A 6.1a, what happens if the predator population goes to zero? Try to give both a qualitative and a quantitative answer.

Question 6-3. Plot the equation you derived in the answer to Question 6-2 for $t = 1$ to 10, assuming $a = 1$ and $x_0 = 1,000$ (you should get something like Figure A 6.3 below). What happens to the population as t gets large? Try this for various values of a and x_0, and different ranges of t.
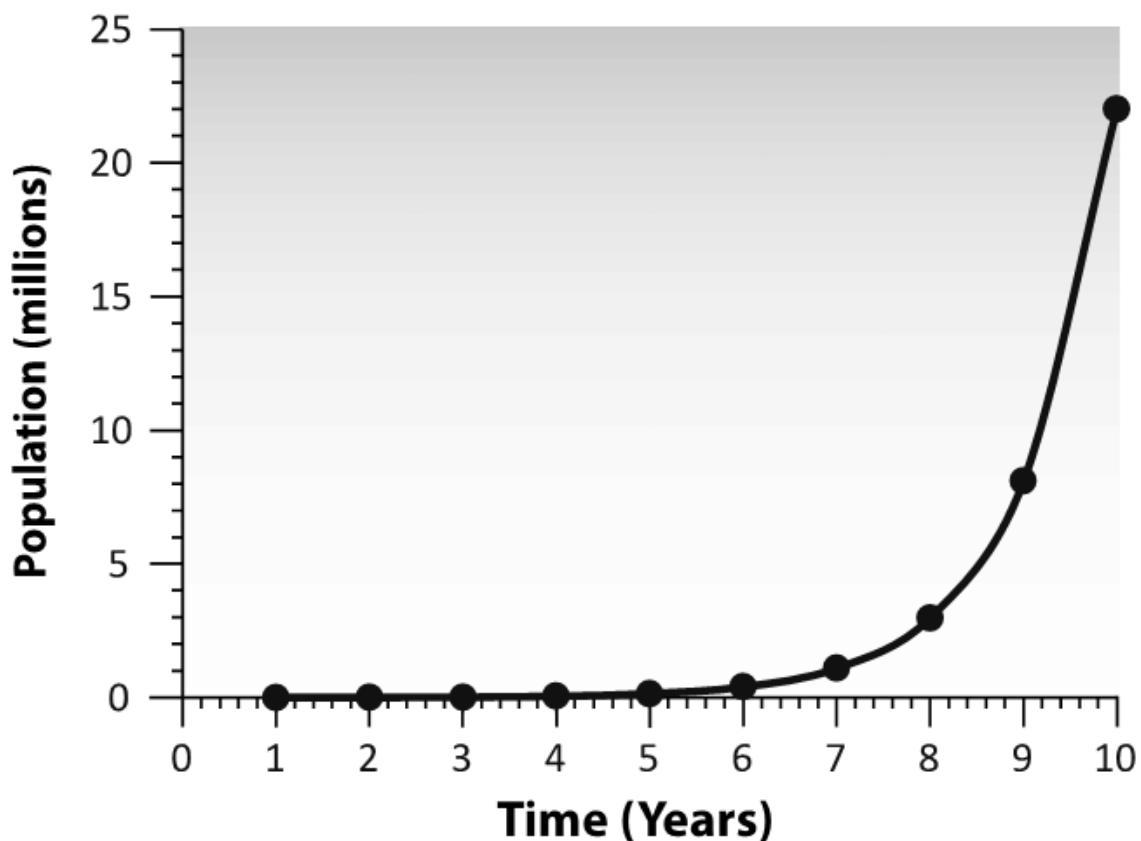
Figure A 6.3. An example of exponential population growth.

Question 6-4. Discuss how your results for questions 6-1 and 6-2 might apply to humans (hint: compare Figure A 6.3 to Figure 8.13). Discuss long term implications.

Question 6-5. The predator-prey model has been applied to a number of systems that experience boom and bust cycles, not only in biology, but also in economics. Discuss how a predator-prey model might apply to the two-component system of oil price and the supply of oil drilling rigs.

Question 6-6. Discuss how similar boom and bust cycles apply to commodities such as copper and zinc.

Chapter Seven

Subsidence, flooding, and failure of the MRGO levee during Hurricane Katrina. Subsidence of the land surface over time due to compaction or oxidation of soil organic matter can be modeled as an exponential process, where the process is fast at first and then slows down (many natural processes follow such behavior). Figure A 7.1 shows one such model that may apply to parts of New Orleans. This model explains why today's lowest lying areas (as much as 3 meters below sea level) actually have very low subsidence rates, as observed in the present-day subsidence rate and elevation data for New Orleans (*Dixon et al.,* 2006). In other words, many low-lying areas got that way by subsiding quickly for several decades, until they reached a nearly stable, low elevation state. Basically, you can only compact so far. The amount of compaction since deposition depends on time, the type of sediment, and the original thickness of young, compactible sediments.
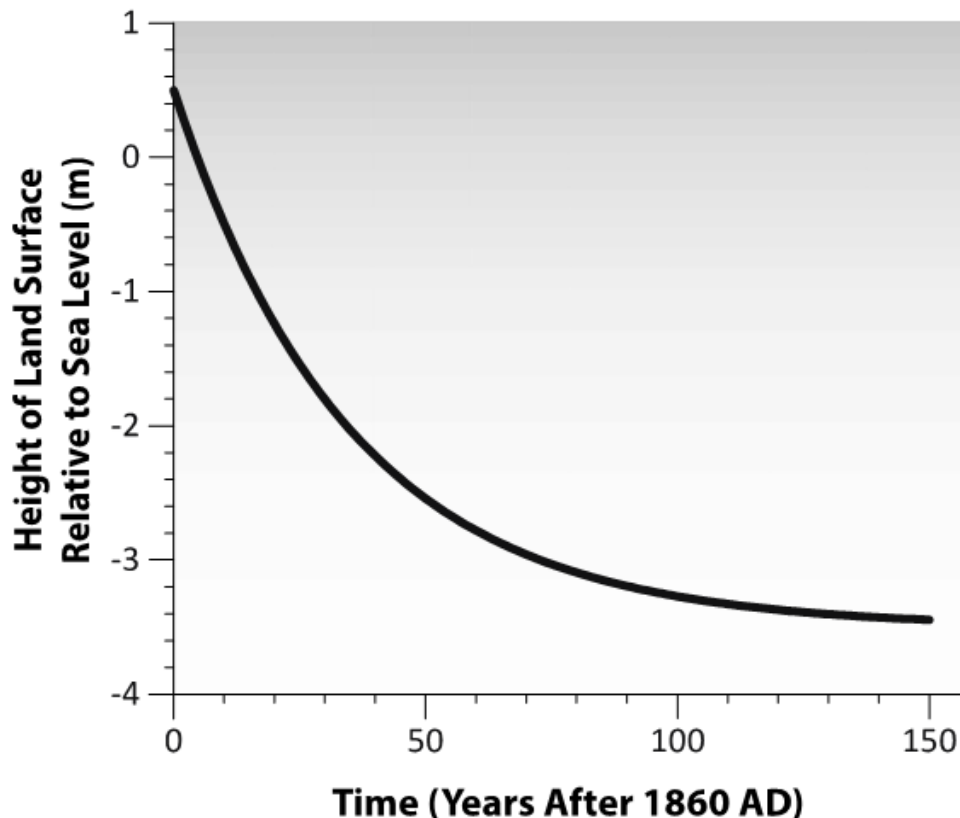

Figure A 7.1 Subsidence of the land surface over time, assuming it is caused by an exponential process of oxidation and compaction. The land surface is assumed to be 0.5 meters above sea level at Year 0 (e.g., 1860 AD, about the time when levees were first constructed in New Orleans) and then subsides at a rate given by the equation:

$$z(t) = z_0 e^{-t/\tau} - z_1$$

where $z(t)$ is the height of land at any time after Year 0, t is time in years, z_0 is the total subsidence after infinite time (assumed in this case to be 4.0 meters), z_1 is given by z_0 minus the initial height (0.5 meters), and τ is an exponential time constant, set equal here to 35 years. z_0 relates to the original thickness of compactible sediments, and τ relates to the material properties. In this model, rapid subsidence is observed in the first few decades, after which the rate of subsidence slows considerably. Most subsidence is finished after 75-80 years, when the surface is about 3 meters below sea level. For example, at Year 30, the average subsidence rate (the slope of the tangent to the curve) is about 50 mm/yr. By Year 100, the subsidence rate is only a few mm/yr.

Question 7-1. The subsidence model described in Figure A7.1 (above), combined with available data on subsidence rates, can be a useful forensics tool to assess failure of some of New Orleans levees during Katrina. One levee that failed was adjacent to the MRGO canal. Failure of this levee, initially constructed in wetlands in the 1960's, contributed to extensive flooding of St Bernard's Parish and the Lower 9th ward, east of downtown New Orleans. Assume that a subsidence model modified slightly from that described above, with z_0=3.0 meters and a different value of τ, applies to the MRGO levee and the land beneath it. If the subsidence rate measured between 2002 and 2005 is 20 mm/yr, derive a model that allows you to estimate the total amount that the levee subsided between the time of its construction in 1960's and the onset of Hurricane Katrina in 2005.

The detailed answer to this question is given at the end of this Appendix. You should obtain a total amount of subsidence after 40 years of approximately 2.6 meters (8.5 feet). This would put the levee well below its initial design height (levees are normally constructed to withstand storm surge in the range of 15-20 feet). Loss of 8.5 feet of elevation would likely allow over-topping of the levee during a hurricane, and subsequent rapid erosion and failure. Although many different models could be constructed consistent with a subsidence rate of 20 mm/yr 40 years after construction, it's very difficult to find any plausible model that results in total subsidence less than about 5 feet. It seems clear that at least some of the levees were too low, and no one had bothered to measure their heights properly, or if they had, sound the alarm and get them fixed. That mistake contributed to the more than $100 billion (US) cost of Hurricane Katrina, and 1800 lives lost.

Question 7-2. A recent article in *Scientific American* (Marshall, 2015) described the plight of residents of Tangier Island, a small island in the southern part Chesapeake Bay in the US state of Virginia. The island was first settled by English immigrants in the late 1600's, and has been continuously occupied at least since 1700 AD. The island has lost more than two thirds of its area since the mid-1800's. Sea level rise has made much of the island uninhabitable, and will likely force the last residents to leave within the next 50 years.

The *Scientific American* article stated that the cause of the flooding was a combination of sea level rise from global warming, and local effects from slowing of Atlantic Ocean circulation, also probably due to global warming. The impact of unusual currents is difficult to quantify, and is the subject of on-going research by the scientific community. So it's difficult to know how significant that factor is. But it is possible to quantify two other processes, one not mentioned in the Scientific American article: glacial isostatic adjustment (GIA; see Figure 7.9). Using a recent estimate of the rate of global sea level rise for the 20th Century (Hay et al., 2015), and a recent estimate of land subsidence from GIA at this location (Karegar et al., 2016), estimate the relative contributions of global sea level rise and GIA to the problems being experienced by Tangier's residents, for the time period 1700 to 2015. Which factor is more important? Assume that the 20th Century estimate applies from shortly after the industrial revolution to 1990. After 1990, assume that global sea level is rising at the rate given in Figure 7.4

The important "take away" message for this problem is that while global warming and its effects are important issues, in detail the "early indicators" (impacts we observe today) are often more complex – it is the sum of various processes, not just those related to global warming, that are likely to cause some of our near-term problems. Many of these processes are "boundary conditions" for living on planet Earth – we can't change them.

Question 7-3. A global warming skeptic decides to use the results of Question 7-2 to argue that sea level rise is mostly natural, and not caused by global warming. How would you answer this person?

Chapter 8

Without an atmosphere, Earth would be a pretty inhospitable place, and not just because we would lack oxygen to breath. The moon receives roughly the same amount of solar radiation as earth, but lacks an atmosphere to moderate the climate. On the night side, surface temperatures fall as low as about -230°C (-184°F). On the daytime side, surface temperatures rise as high as about +120°C (+248°F). The greenhouse effect is a key part of an atmosphere's ability to moderate temperature, and "smooth out" the highs and lows. It is critical to our survival.

Although John Tyndall first pointed out how the greenhouse effect works in the mid-19th century, a full understanding of the process and CO_2's role had to await discoveries in atomic physics in the early part of the 20th century. Max Planck's theory of the quantized nature of energy helped to clarify why greenhouse gases, even if present in trace amounts, are so efficient at absorbing energy from the sun. Planck won the Nobel Prize in physics for his work in 1918.

Any warm object radiates energy, and the amount and wavelength distribution of that energy is temperature-dependent. Hotter things give off shorter wavelength radiation

compared to cooler things: a heating coil on an electric stove top that is bright orange is hotter than one glowing dull red, and orange light has a shorter wavelength than red light. Planck discovered the non-intuitive result that you can't have just any old value of energy; it comes in discrete packets, or "quanta". It's a bit like money, quantized by its smallest unit (e.g., one cent, representing one hundredth of a dollar in the US, or one hundredth of a Euro in the 18 countries of Europe currently in the Euro zone). If you buy a loaf of bread in the US or the Euro zone, the price might be 1.99 or 2.99 (in units of the local currency) but is unlikely to be listed as 1.995 or 2.995, because half cents don't exist for these currencies.

When the Earth is irradiated by the sun, both the atmosphere and the surface warm up, but for reasons that differ from your typical greenhouse, reasons that are closely related to Planck's discoveries. For the most part, Earth's lower atmosphere is not heated very much by direct solar radiation. Most of the sun's output is in the visible part of the spectrum, and the atmosphere is relatively transparent to these wavelengths – the Sun's photons pass through without much interaction. That's one reason why we see in the visible part of the spectrum – our eyes evolved to take advantage of all the energy in this spectral band. For the atmosphere to heat up, something has to be capable of absorbing at least some of the Sun's energy.

Certain molecules in the atmosphere *are* quite good at absorbing energy from photons, if those photons happen to have the right energy. One of Planck's discoveries was that the energy of a photon is inversely related to its wavelength (longer wavelength means less energy). The key energy range for warming the atmosphere is in the infrared part of the electromagnetic spectrum, with longer wavelengths compared to visible light. The key molecules turn out to be those with odd numbers of atoms, such as CO_2 (carbon dioxide, with one carbon and two oxygen atoms), H_2O (water, with one hydrogen and two oxygen atoms), O_3 (ozone, with three oxygen atoms) and CH_4 (methane, or natural gas, with one carbon and four hydrogen atoms). These molecules have a complex set of properties (rotational, vibrational, and bending motions) with discrete energy levels, allowing them to absorb photons of specific wavelengths (with the same discrete energies), mainly in the infrared parts of the electromagnetic spectrum. Having absorbed this energy, the molecules are now in an excited state, bouncing and vibrating; the atmosphere has absorbed energy and heats up. Eventually the excited molecule will re-radiate the photon, and return to its "ground" (un-excited) state. But even then there is a good chance that the photon and its energy will not be lost to space, but rather be re-absorbed by another nearby greenhouse molecule. The directions of the re-radiated photons are random, and only a few possible pathways lead directly to space; most lead to other absorptions within the atmosphere, or back to the ground. You can see that the chances of this are increased if there are more greenhouse molecules. That's why the whole process is so sensitive to the amount of carbon dioxide – a little bit goes a long way.

So the question is, if the atmosphere is so good at absorbing infrared radiation, but the sun puts out mainly visible radiation, where does the infrared radiation come from? It turns out that much of it comes from the Earth itself, and involves a radiative balancing act. This balancing act determines Earth's average temperature, and depends on the

amount of CO_2 and other greenhouse gases in the atmosphere. In qualitative terms, visible light from the sun passes through the atmosphere, heating the Earth's surface. The Earth's surface then emits longer wavelength infrared radiation, some of which is absorbed by the atmosphere and re-radiated back to Earth, while the rest is lost to space. The concept is illustrated schematically in Figure A 8.1. On average, the amount of energy that Earth receives from the Sun is balanced by energy radiated back to space. If for some reason Earth happened to receive extra solar radiation, its surface would heat up, leading to an increase in surface temperature and increased outgoing radiation to maintain balance. The remaining parts of this section describe the process in more quantitative terms, including a series of questions that show how radiative balance is actually achieved, how increasing CO_2 in the atmosphere changes the balance, and an example consequence.
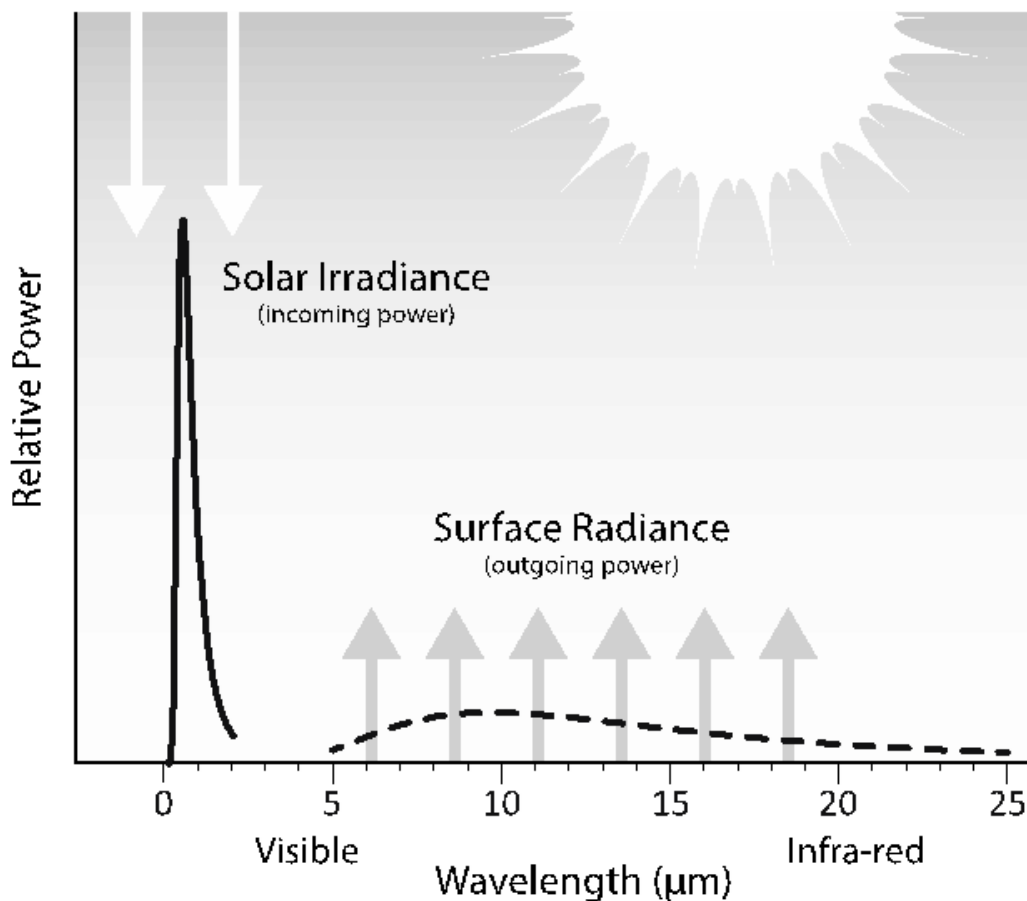

Figure A 8.1. Incoming solar radiation in the visible part of the electromagnetic spectrum compared to outgoing radiation emitted by the Earth into the atmosphere in the longer wavelength infra-red part of the spectrum, based on equation A 8.1. On average, there is a balance between incoming and outgoing power.

Planck's law describes the distribution of radiation emitted by any body over a range of wavelengths as a function of the body's temperature:

$$S(\lambda) = \frac{2\pi hc}{\lambda^5} \frac{1}{e^{ch/\lambda kT} - 1} \qquad\qquad (A\ 8.1)$$

where S is the emitted power in Watts (W) per square meter per unit wavelength λ (also in meters), $\pi = 3.14$, h is Planck's constant (6.6 x10^{-34} Wsec^2), c is the speed of light (3*10^{8}m/sec), k is the Boltzmann constant (1.38x10^{-23} Wsec/°K), and T is the absolute temperature in °Kelvin (°C+273). If you set the temperature in equation A 8.1 to 6,000°K (roughly the average surface temperature of the Sun) you would get a curve that describes the amount of incoming solar radiation in terms of its wavelength distribution (similar to the curve on the left hand side of Figure A 8.1). The actual amount of received solar radiation depends on a planet's distance from the Sun, and decreases as $1/r^2$, where r is distance.

If we take the first derivative of equation A8.1 and set it to zero, we get an equation that describes the wavelength of maximum emittance, known as Wien's law:

$$\lambda_m = \left[{}^a\!/_T\right] \qquad\qquad (A\ 8.2)$$

where $a = 2898$ μm°K (in other words, if T is in units of °K, then the wavelength will come out in units of microns, μm, or one millionth of a meter).

Question 8-1. Assume the average temperature of the sun's surface is about 6,000 °K, while the average temperature of the Earth's surface is about 288°K (15° C). Using Wien's law (equation A8.2) calculate the wavelength in microns of the peak radiation emitted by these two bodies. What are these two parts of the electromagnetic spectrum called?

The total amount of energy per unit time (power) radiated by a hot or warm body such as the Sun or Earth winds up being an important quantity. These values can be derived by considering the integral of equation A8.1, representing the mathematical operation equivalent to adding up all the energy in the various "slivers" of wavelength:

$$S = \int_0^\infty S(\lambda)d\lambda = \sigma T^4 \qquad\qquad (A\ 8.3)$$

where $\sigma = 5.67 *10^{-8}$ W/m^2 °K^4 is the Stefan-Boltzman constant, and the equation is called the Stefan-Boltzman law. In the case of the Sun illuminating Earth, the radiation received at the top of the atmosphere has been measured by satellite, and has a value (S_0) of about 1366 W/m^2. It can also be derived from equation A8.3, after accounting for the geometric losses (spherical spreading) between the Sun and Earth.

Question 8-2. The planet Mercury, 57.9 million km from the sun, receives about 9,130 W/m^2 of solar irradiance. Earth's average distance from the sun is about 149.6 million km (it varies by about 5 million km over the course of a year). Use these values to derive the solar irradiance received by the Earth. How does it compare to the satellite-measured value?

Your answer to Question 8-2 represents the amount of solar radiation received at the top of the atmosphere. To figure out how much of this flux is available to heat the Earth, there are several geometric and other effects that must be considered. For example, at any one time, only half the Earth (the day-time side) is receiving this solar flux. Also, only areas near the equator are "aimed" directly at the sun and receive the full flux; areas near the pole receive very little direct radiation because of their orientation. The next question asks you to account for these geometric effects and calculate something we'll call the effective solar irradiance.

Question 8-3. Calculate the effective solar radiance received by the Earth (Hint: consider the average over 24 hours) (Second hint: assume the Earth receives 1369 W/m^2 at the top of its atmosphere from the Sun.

There is one other effect we need to consider. A certain fraction of the solar irradiance received by Earth, termed the albedo, is reflected directly back into space, from clouds and particles in the atmosphere. Snow and ice at the surface also contribute to albedo. On average, about 30% of incoming radiation is reflected (i.e., albedo = 0.3) meaning that 70% (239 W/m^2) is available to heat the Earth.

If the amount of sunlight reaching the Earth remained constant, and the amount of greenhouse gases in the atmosphere also remained constant, then incoming energy from the sun would be balanced by energy radiated from the Earth to space. In this case, Earth's average surface temperature would remain constant.

To illustrate this, we'll first calculate the temperature of the Earths surface if there were no greenhouse effect (Question 8-4), then add in the greenhouse effect to see the difference (Questions 8-5 and 8-6).

Question 8-4. Using an energy balance approach, estimate the average surface temperature of the Earth, ignoring the greenhouse effect. Assume the incoming solar radiation received at the surface of the Earth, S_s, is 239 W/m^2.

An approximate expression for Earth's average temperature in the presence of the greenhouse effect, again using the energy balance approach, but allowing the atmosphere to participate in the balance, is:

$$T_S = \left[\frac{2S_s}{\sigma(2 - \alpha)}\right]^{1/4} \qquad\qquad (A8.4)$$

where α represents the absorptivity of the atmosphere, varying between 0 and 1. When $\alpha = 0$, there is no greenhouse effect, basically what we assumed in question 8-4. When $\alpha = 1$, the atmosphere absorbs all of the radiation emitted by the Earth (maximum warming).

Question 8-5: Use equation A8.4 to estimate Earth's average surface temperature, assuming values for α of 0, 0.5, and 1.0.

Question 8-6. In the last 100 years, CO_2 in the atmosphere has increased by about 30%. Assume that this causes a corresponding increase in the atmospheric absorptivity α from 0.70 to 0.75. Calculate the corresponding increase in Earth's average surface temperature during this period.

Of course, the models used to estimate Earth's average temperature in Questions 8-4, 8-5 and 8-6 are greatly over-simplified, but they do illustrate important concepts. One process they ignore is feedback. Feedbacks in the climate system are not fully understood, and make future predictions challenging. Feedbacks probably give climate scientists their worst headaches, as they are tough to model – small changes in model inputs can make big differences in model outputs (climate predictions).

Feedbacks can be either negative or positive. A negative feedback promotes stability – an increase in warming induces some process that decreases atmospheric CO_2, decreasing future warming. A positive feedback is the opposite - a small increase in warming promotes some other process which leads to increased atmospheric CO_2, which in turn leads to more warming. Positive feedbacks can lead to instability - there is a chance that some sort of critical "tipping point" will be exceeded, producing large future changes. One example of a positive feedback is the role of Arctic tundra and permafrost. Increased summer warming of the tundra will lead to melting of permafrost, which leads to emissions of CO_2 and CH_4 (methane) which increases the greenhouse effect and leads to more warming. Arctic warming and consequent changes to the tundra are a possible tipping point in the climate system.

Water vapor has both positive and negative feedbacks. On one hand, increased warming leads to greater evaporation over the oceans, which leads to more clouds, which increases the atmosphere's albedo, which leads to cooling. On the other hand, water vapor itself is a powerful greenhouse gas, which increases warming (models suggest these effects largely cancel, although there is some uncertainty). The role of aerosols (small particles in the atmosphere) is similar. Aerosols from increased industrialization and air pollution may increase the atmosphere's albedo (leading to cooling) but soot (sometimes called black carbon), a major component of aerosols from coal-fired power plants and burning of wood, will absorb more solar energy (leading to warming). Current thinking on aerosols is that these effects also cancel out, or nearly so.

One worrisome positive feedback involves warming and terrestrial carbon sinks (things that store carbon on land, especially forests). Increased warming could lead to mass die-offs of forests, from a combination of drought, forest fire, and migration of insect pests into more northerly regions, where existing trees are not well adapted for resistance (basically, the bugs migrate faster than the trees). This has the potential to unlock vast stores of CO_2 over a relatively short time. Some forest ecologists believe we are already starting to see such changes in western North America and Russia. On longer time scales, vegetation will presumably adapt to changing conditions, but the possibility of rapid short-term changes has the potential for significant climate and ecosystem disruption.

The next question asks you to look at one of the consequences of our changed energy balance. It highlights an important data set that shows pretty clearly that Earth has warmed significantly in the last 100-200 years. It is a key piece of evidence in support of the concept of human-induced climate warming, but one that is often overlooked: how much has the surface of the solid Earth warmed? Although most people focus on temperature when discussing warming trends, it is also useful to look at heat content, which incorporates the idea of heat capacity (the ability of an object to store heat or change temperature with a given amount of heating). The change in heat content of an object is given by its mass, times its heat capacity, times the temperature change. Measurements and calculations suggest that the heat content of both the atmosphere and ocean has increased significantly in the last few decades (Table A8.1). Scientists interested in global warming usually focus on the atmosphere and ocean because they are mobile and contribute the most to weather and climate. Note that the ocean holds a lot more heat than the atmosphere.

Table A8.1: Change in heat content for the atmosphere and ocean*

Atmosphere	0.7×10^{22} J
Ocean	18.2×10^{22} J

*Data from Levitus et al. (2001) for the period 1950-2000.

The imbalance in Earth's energy budget also affects the heat content and temperature distribution of the upper few hundred meters of the solid Earth. Temperature profiles in mines and deep boreholes have been used by geophysicists to understand Earth's inner heat sources, the driving force for plate tectonics, and ultimately, its earthquakes and volcanoes (see Chapter 2). Other things being equal, we expect a more or less linear increase in temperature with depth because of these inner heat sources, with generally cooler temperatures at Earth's surface giving way to hotter temperatures at deeper depths (there is a small amount of variation in the upper few meters reflecting winter-summer variation, but below this the variations average out). Oil pumped from several km depth is quite hot – the Alaska pipeline which carries oil southward from the Prudhoe Bay oil

field on Alaska's north slope had to be insulated so that heat from the oil does not melt the permafrost upon which the pipeline sits.

In a paper published in 1982, Dr. Arthur Lachenbruch and colleagues at the US Geological Survey showed that temperature profiles in deep boreholes in Alaska were skewed to the warm side near the surface of the Earth (Figure A 8.2) by amounts that could best be explained by recent warming.
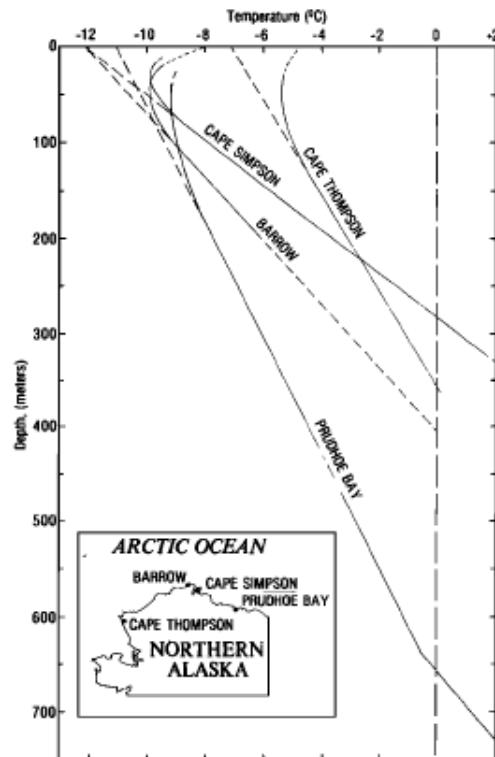

Figure A 8.2. Profiles of temperature versus depth for several boreholes in northern Alaska. The dashed lines near the surface are the expected linear distribution. Instead, most of the profiles are curved towards warmer surface temperature. From *Lachenbruch et al.* (1982).

 In their paper, they stated:

"…the departure of the generalized temperature profiles … from linearity in the upper 200 m is the effect of a systematic change in the heat balance at the earth's solid surface during the last 100 years or so."

 The authors went on to conclude that the average warming during that period was about 1.8°C. Later work by other scientists showed that anomalous temperature profiles in deep mines and boreholes were common, were found across most regions of the Earth, and were consistent with significant warming over the last 100-200 years. However, the anomalous temperatures in most areas were typically less than those found in the original Alaska study, in the range 0.8°C – 1.0°C. We now understand that this reflects the faster rate of heating in the Arctic compared to other regions. Lachenbruch and colleagues's

1982 publication should have been a wake up call for scientists, the media, politicians, and the public that global warming was a clear and present danger. Instead, it was largely ignored.
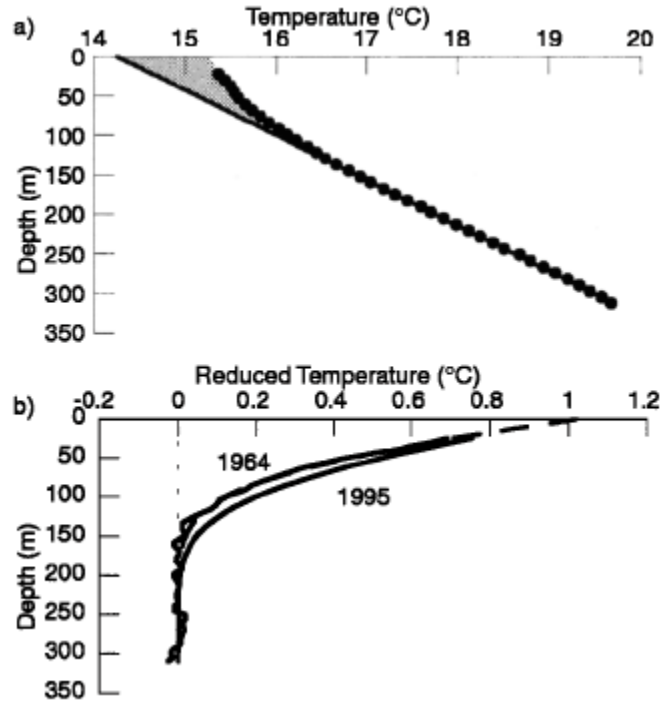

Figure A8.3, top: Example mid-latitude temperature profile (small circles = data, line shows linear extrapolation to surface, shaded area shows region of anomalous heat). **Base**: reduced profile (observed temperature minus expected temperature), highlighting the upper 150 meters where temperatures are warmer than expected from the linear model. From *Harris and Chapman* (2001).

Question 8-7. Calculate the anomalous heat content in the upper crust of the solid Earth associated with modern global warming. How does it compare to the values listed in Table 1? Use the data summarized in Figure A8.3, and assume that it adequately approximates average heating across the planet.   Hint: if you work in SI units (kilograms, meters, °C) the answer will come out in Joules, for easy comparison to Table A8.1.

Impacts on Human Health. In Chapter Seven, I pointed out that some disasters have multiple causes; global warming may be just one of several contributing factors. Coastal flooding from hurricane-related storm surge has always occurred, but may be increasing in frequency due to a combination of factors, including coastal subsidence,

increased population density in coastal areas, sea level rise, and increased hurricane intensity associated warming ocean water. The last two factors are clearly related to global warming, but subsidence is not. Recent flood disasters have tended to occur on low-lying river deltas and subsiding areas – these events are "canaries in the coal mine", clear early indicators of future flood events that will strike over broader regions as the effects of global warming intensify. It would be wrong to focus on the role of subsidence in New Orleans' Katrina disaster, and then claim that since global warming was not the major factor, there are no lessons to be learned.

The human body tries to maintain its internal temperature close to 37 ° C (98°-99°F). We have cooling mechanisms that work pretty well with external temperatures up to about 41°-43° (106-109°F), depending on humidity, length of exposure, and rate of exertion. Much higher than that, and our bodies may have trouble coping. Some parts of the globe are now reporting daytime highs in excess of 49°C (120°F).

In the last few decades, agricultural workers in low-elevation parts of Central America, India and Sri Lanka have been dying at higher than expected rates from kidney failure (sometimes called CKD, for chronic kidney disease). These workers are exposed to significant heat stress during the day, and it is possible that temperatures have risen past a critical threshold in some parts of the globe, for some outdoor workers. But other explanations are possible, including exposure to pesticides, labor practices (long hours, few breaks, limited supplies of clean, fresh water) and cultural preferences (sugary soft drinks have become popular in many parts of the world, and can add to kidney stress). It is also possible that a combination of two or more of these factors is responsible. Chronic dehydration is likely an important cause, hence the last two factors, unrelated to global warming, are clearly relevant.

Question 8-8a. Read the articles by *Weiner et al.* (2012) and *Chatterjee* (2016) referenced at the end of this section, and list all the possible causes of CKD that you can think of.

Question 8-8b. List the factor you think is most likely responsible. Briefly describe the evidence in favor of your hypothesis.

Question 8-8c. How would you go about testing your hypothesis?

Question 8-8d. Assuming that global warming is not the major factor in tropical CKD, could it be a contributing factor? Why or why not? If it is a factor, what lessons can we draw from this tragedy?

Question 8-8e. Based on the discussions above, what advice could you give to agricultural workers in tropical countries and other worker exposed to high levels of heat stress to minimize their risk of developing CKD?

The long long term. One theme of this book is the importance of considering long-term trends, and long time series, in order to properly characterize a given hazard. We initially considered long-term as anything longer than a typical human time span, but then found that for events such as earthquakes (Chapters Three, Four and Five; Questions A 3-1 and A 3-2), we needed to consider at least several thousand years. For global warming and climate change, we compared temperature records of the last 100 years (Figure 8.3), 250 years (Figure 8.4) and one thousand years (Figure 8.5). Only the one thousand year record definitively shows human impact on climate. However, Figure 8.12 (more than 10,000 years in length) shows human impact most clearly.

One difference between global warming and earthquakes is that human activities are disrupting the climate in ways we don't fully understand. Hence the past record of climate change is not necessarily an accurate guide to what may happen in the future. In contrast, human activities are unlikely to change the occurrence of large subduction zone earthquakes, the real "killers" in terms of hazard because they can produce giant tsunamis (discussed in Chapters 3, 4 and 5). Activities such as fracking, where fluids and muds are injected into the subsurface to produce natural gas, may temporarily stimulate moderate size earthquakes near the injection site, but fracking as currently practiced is unlikely to alter the pattern of subduction zone earthquakes and tsunamis. Thus we can study the past, get some idea of the frequency of these events and their destructive capacity, and then use this understanding to better prepare for future earthquake and tsunami disasters via improved design, engineering and construction.

The past history of climate change is not necessarily a reliable indicator of what the future holds because human activities are changing conditions at an unprecedented rate, modifying some of the key factors that affect climate. These include rapid increases in carbon dioxide (CO_2) and methane (CH_4) in the atmosphere (which increases warming), rapid reductions in the amount of tree cover on the Earth's surface (which reduces the planet's ability to remove excess carbon from the atmosphere), and rapid increases in the acidity of the oceans (which will likely impact carbon cycling in the oceans in ways we do not fully understand). The speed of these changes has no precedent in the recent geological past.

Even so, "going long" can improve our understanding of key climate processes. For example, looking at past times when conditions were changing might help us understand today's rapid changes, even if past changes were not quite as fast. But how far back should we go? In questions A 3-1 and A 3-2, I suggested that we needed to cover at least the last three earthquake cycles to get a reasonably good idea of how a given fault system is behaving. How long is a climate cycle? And how do we go back in time? Much like the techniques used in paleoseismology (Chapters Four and Five), paleoclimate scientists have had to infer cycles of past climate change using a variety of stratigraphic and chemical techniques in sediment deposits and ice cores. In the latter, the ice traps bubbles of atmosphere at the time of formation, and is then buried by next year's snowfall, which eventually turns into next year's ice layer, preserving an annual sequence of atmospheric composition.

Figure A 8.4 shows a record of temperature and atmospheric CO_2 going back 800,000 years, using a variety of paleoclimate techniques. Several cycles of warming and cooling are apparent, each about 100,000 years in length.
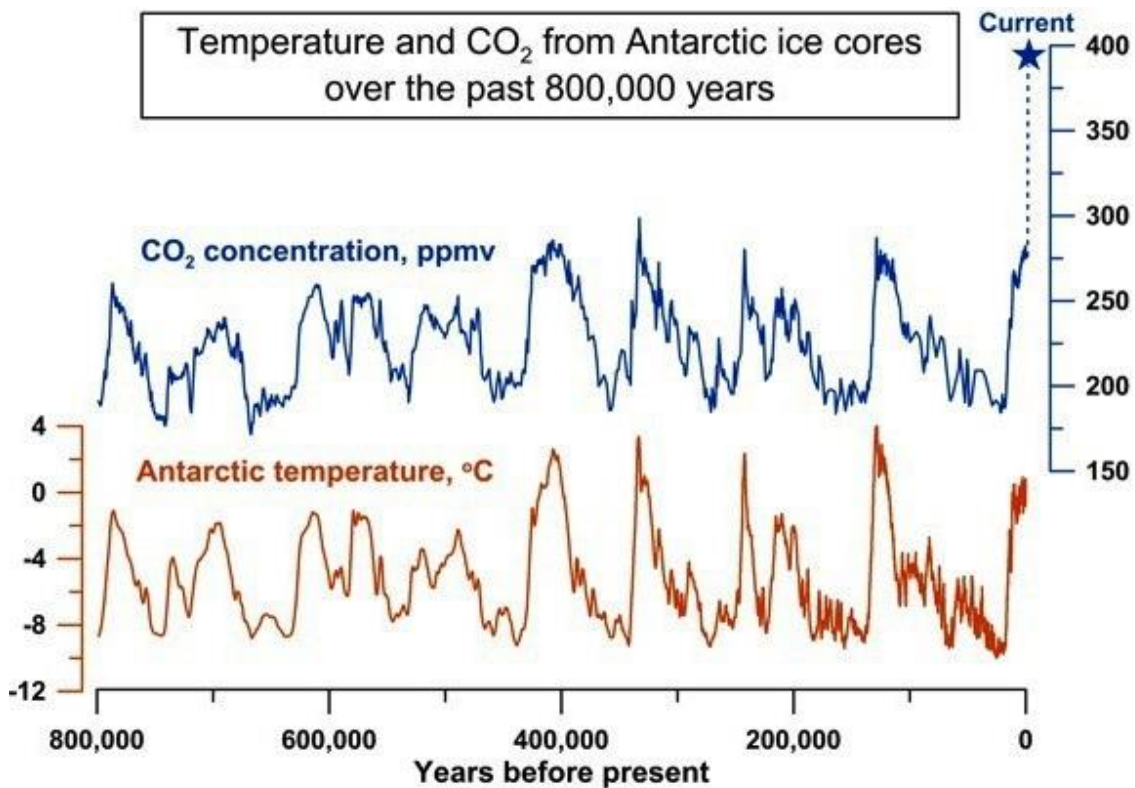

Figure A 8.4. Temperature and atmospheric CO_2 spanning the last 800,000 years, using data from Antarctic ice cores. Figure courtesy of A. Shevenell, using data from *Luthi et al.* (2008) and *Shakun et al.* (2015).

Climate scientists call this the glacial – interglacial cycle. It is paced by long-term changes in Earth's orbit around the sun, which varies in a predictable way. Over hundreds of thousands of years, the amount of energy we get from the sun changes by a small amount, enough to influence Earth's climate system. The cycles are called Milankovich cycles, after Serbian scientist Milutan Milankovich (pronounced Mil-AN-kovitch) who first discovered them. Inspection of Figure A 8.4 suggests that if past trends continue, the next glacial maximum (ice age) would be at least 15,000 - 20,000 years in the future.

The apparent simplicity of Figure A 8.4, and the explanation above, hides some puzzling aspects. For example, why is warming much faster than cooling (the peaks and valleys are not symmetric)? Also, why does CO_2 vary in lock step with temperature, since we can't appeal to earlier industrial revolutions to change CO_2 on these time scales? It turns out that the changes in solar insolation (the scientific term describing how much energy we get from the sun) associated with the Milankovich cycles are probably too small, by

themselves, to cause the observed temperature changes. The best explanation involves feedbacks, explored in the next question.

Question 8-9. Speculate on possible causes for the correlation between temperature and CO_2 illustrated in Figure A 8.4.

Tundra feedback has already been described. Small insolation-induced increases in temperature (especially in the summer) lead to melting of permafrost, releasing methane (CH_4) that was previously sequestered in a frozen state in the tundra. This increases warming, since methane is a powerful greenhouse gas. The methane eventually reacts with oxygen to form carbon dioxide and water.

Methane is also present below the seafloor, in most of the world's continental shelf sediments, sequestered in a frozen, ice-like substance called gas hydrate. As the oceans warm, gas hydrates start to melt, potentially leading to large methane releases, further increasing atmospheric temperatures.

Albedo changes in the polar regions represent another positive feedback. Small amounts of warming lead to reductions in sea ice, reducing the albedo of the oceans (ice is white, reflecting solar radiation back to space, while ocean water is dark, reflecting much less solar radiation). Small increases in summer warming can also lead to a reduction in the area of ice sheets such as Greenland, increasing the area of dark rock. Earlier spring snow melt across the Arctic will reduce the region's average albedo. While not leading directly to increased CO_2, albedo changes could amplify the methane feedbacks discussed above.

A feedback involving volcanoes has long been postulated, most recently by Drs. Peter Huybers and Charles Langmuir at Harvard University. When volcanoes erupt, they release large amounts of CO_2 into the atmosphere. At the present time, industrial and transportation sources dwarf volcanoes as a source of CO_2, but prior to the industrial revolution, volcanism represented a major source. Many of the world's volcanoes are capped by glaciers. With warming, the glaciers retreat, reducing the weight of ice on top of the volcano. Much like popping the top off a champagne bottle, the reduction of weight on the top of the volcano can stimulate an eruption, releasing large amounts of CO_2.

Methane analyzed in ice cores shows a pattern similar to that displayed by CO_2 (Figure A 8.4). This similarity suggests that feedbacks involving methane (e.g., tundra melting or release of continental shelf methane from gas hydrates) are probably an important feedback mechanism in the climate cycle.

Question 8-10. Discuss how feedbacks may be responsible for the asymmetric (sawtooth) pattern of cycles in Figure A 8.4 (rapid warming, slow cooling).

One of the things that worries climate scientists is the magnitude of positive CO_2 feedbacks, how they will interact with our own injections of CO_2 into the atmosphere, and the resulting long-term impact on Earth's climate and ecosystems. Natural feedbacks have led to past conditions that allowed crocodiles to live near the poles, sea levels much higher than those we see today, and temperatures near the equator that may be too high for humans to survive. Even so, there are natural buffers in the system (still not completely understood) that in the past have prevented even more extreme conditions. Natural levels of CO_2 have not exceeded 300 ppm for at least the last 800,00 years (Figure A 8.4). Now with our industrial activities, CO_2 concentrations have hit 400 ppm, and will soon rise to 450-500 ppm. What happens when we change and amplify the CO_2 feedback process? What happens if the rate of CO_2 increase greatly exceeds past natural rates? Could we blow through the buffers? The frightening fact is that we don't understand all the long-term consequences, and may not be able to control them.

Answers to Questions

2-1. Long term here means averaged over the last 5,000 years, the age of the stream. This age is consistent with the stream size – large rivers can be millions of years old but small streams are often rather ephemeral features. The San Andreas is a strike slip fault (Figure A 2.1, base) so its motion is purely horizontal. With these assumptions, the slip rate can be approximated by the amount of offset (150 meters, or 15,000 cm) divided by 5,000 years: 3 cm/yr.

2-2. 30 mm/yr (=3 cm/yr), reflecting the difference between -15 mm/yr and +15 mm/yr. Note the similarity between the "long term" rate (averaged over 5,000 years) calculated in the first question, and the "far field" rate (averaged over about 10 years) calculated here. The agreement is not a coincidence, and is a consequence of Reid's elastic rebound theory for earthquakes.

2-3. We will assume that energy release is linearly related to fault rupture area, and just compare areas. For the strike slip fault this area is 1,500 km^2 (100 km x 15 km). For the thrust fault, the area will depend on the dip angle. For a 10° dip and the same maximum depth (15 km), this gives a distance of 86 km in the down direction (15 km divided by the sine of 10°), and an area of 8,600 km^2 (100 km x 86 km), nearly a factor of six bigger. If you have trouble visualizing this, think about cutting a two by four in half, perpendicular to its length. If you cut the board at right angles, the area of the saw cut will be 8 square inches (2 inches times 4 inches). But if you cut it at an angle, the area of the saw cut will be bigger, and the cutting more difficult. Make the angle shallow enough, and it will take a long time to finish the cut.

3-1. To sample this adequately we have to catch all the "peaks" (every 200 years), sample some of the intervening "valleys", and verify that the earthquakes occur with some regularity by going back at least three cycles. So our minimum record should be six hundred years in length, with age dates at least every 100 years, and we should try to

observe a minimum of three events, i.e., a minimum of 600 years. In the absence of historical records this is quite challenging – a single trench location might not yield such a detailed or long record. Geologists often have to satisfy themselves with fewer data. However, if radio-carbon or other method of age dating is available, it may be possible to piece together an equivalent record by correlating events in several trenches, each spanning different parts of the 600 year time interval.

3-2. This sequence has a long-term period of 1000 years. To see why, imagine the earthquake "clock" starts at zero with the first earthquake. Earthquake #2 occurs at t=200 years, earthquake #3 occurs at t=400 years, after which there is 600 years of no earthquakes. The cycle starts again at t=1000 years (400 years plus 600 years). If we wanted to catch three full periods, we'd need a record spanning at least three thousand years.

4-1. The other fragment would have an atomic weight of 96 (235 plus 1 minus 137 minus 3). Note that while one neutron started the reaction, three were given off. This enables the chain reaction that allows a nuclear reactor to produce power, until the concentration of ^{235}U becomes too low in the fuel rod and it must be replaced. However, because of the large number of unstable atomic fragments (like ^{137}Cs) produced during fission, the fuel rod will remain radioactive and hot for a long time.

4-2. The decay path involving 0.66 MeV Gamma rays is about 20 times more common than the 1.17 MeV Beta particle path, making it the better choice. ^{137}Cs is often measured by counting Gamma ray emissions in the vicinity of 0.66 MeV.

5-1. This is good material for in-class discussion. One way might be to look at the general westward progression of earthquakes on the North Anatolian fault. Another way would be to look at the past record of earthquakes at Istanbul. In each case, it is possible to calculate a mean (best estimate of the recurrence interval) and standard deviation (one measure of uncertainty).

6-1. Determining the main source of a pollutant is important both for mitigation (fixing the problem) and assessing legal liability (paying to fix the problem). Two other common sources of lead contamination are factories that produce or recycle products containing lead, such as lead-acid batteries, the type used in most automobiles, and leaded paint, which was in widespread use until the 1970's. Lead from factories is usually deposited close to the source, so a simple map-based analysis, where factory locations are compared to a map of soil lead like the one in Figure A-6.1, can usually distinguish this type of contamination. If lead-based paint is a major source, this could be more difficult to distinguish from a leaded gasoline source, because both can result in highly dispersed lead pollution. One way to proceed would be to compare the map of lead distribution to major roads, taking into account prevailing winds. A correlation would suggest that leaded gas is the main contributor; lack of correlation would suggest an alternate source, such as leaded paint.

6-2. Common sense suggests that the prey population will grow. Let's see what the math says. This question, or ones similar to it, are common in beginning classes in differential equations, and illustrate the properties of natural logarithms, the number e (~ 2.71, sometimes called Euler's number) and exponential growth.

Setting $y = 0$ in equation A 6.1a, we have:

$$\frac{dx}{dt} = ax$$

For simplicity we drop the subscript on a, since we now have only one constant. Re-arranging and integrating gives:

$$\int_{x_0}^{x} \frac{dx}{x} = a \int_{t_0}^{t} dt$$

where x_0 is the population at the initial time $t_0 = 0$. Solving the integral gives:

$$ln\, x - ln\, x_0 = a(t - t_0) = at \ \ (\text{since } t_0 = 0)$$

where $ln\, x$ is the natural logarithm (usually just called log) of x. From the properties of natural logs this is equivalent to:

$$\ln \frac{x}{x_0} = at$$

Since $e^{\ln x} = x$, we can write this as:

$$\frac{x}{x_0} = e^{at}$$

Multiplying both sides by x_0 gives

$$x = x_0 e^{at} \qquad\qquad\qquad \text{A 6.2}$$

Equation A 6.2 states that the population of x (the prey) will grow exponentially with time from some initial population x_0. Of course at some point this becomes unrealistic (i.e., when t gets large) - eventually the prey runs out of food. In a finite system like the Earth, exponential growth always hits some limit.

6-3. Figure A 6.3 shows equation A 6.2 plotted for $a = 1$ and $x_0 = 1000$. Note the rapid growth in population after just a few years. This is an example of exponential growth. As you try different values of a and x_0, note that population always begins to grow rapidly at some point in time.

6-4. Humans are no longer threatened by large predators (we've killed most of them, or put them in zoos). Until the 19th Century the main limits to human population growth were disease (which functions something like a predator) and food supply. Advances in medicine have greatly reduced the toll from disease, while technological developments in areas such as fertilizers, pesticides, agricultural techniques, and food preservation and distribution have greatly expanded food production. These improvements in medicine and food supply have led to exponential growth in human population (Figure 8.13). Common sense suggests that this cannot go on forever, but the true limits to continued human population growth are not well understood. This issue and related topics such as family planning are controversial in many cultures and religions, and it can be difficult to talk about them objectively.

6-5. The price of oil depends on a large number of factors, and is unlikely to follow the simple oscillatory behavior shown in Figure A 6.1. Nevertheless, we can gain some insight into the large price swings exhibited by this commodity by considering two co-dependent factors, the price of oil and the number of drilling rigs searching for new oil supplies. The price of oil depends on supply and demand, while the number of drilling rigs operating at any one time influences supply, albeit with a time lag. The time difference between when an operator decides to purchase or lease a drilling rig, and when the operation of that rig leads to new oil deposits, can be several years or longer. That time lag contributes to the boom and bust cycle in the price of oil, but it is not the only factor. Consider the following hypothetical scenario.

When the price of oil is low, there is not much incentive for major oil companies or independent operators to search out new supplies or develop new technology. Since existing oil fields tend to decline in productivity over time, an increase in demand, either from population growth or economic growth (the latter enabling more consumers to purchase cars, for example) can lead to an increase in the price of oil (supply does not instantly ramp up). Once the oil price rises high enough, the major oil companies will increase their exploration activity, but initially this does not affect supply, since it takes time for new deposits to be developed. The oil price continues to rise, and now other operators start to jump into the exploration business. This includes "wildcatters", independent operators who may not have been previously involved in the industry, and who are willing to take financial risks because of the possibility of large rewards. New drilling equipment will be ordered to satisfy increasing demand, and new technology may be developed to enable exploitation of unconventional supplies, previously too expensive. All this takes time however, and the price of oil continues to rise. Oil company profits increase, oil company CEOs are hailed as management gurus, automobile drivers sell their gas-guzzling trucks and SUVs and "downsize" to more fuel efficient vehicles, environmentalists proclaim the end of the oil era, and academics write articles about "peak oil".

The steep price rise now promotes even more exploration. Eventually some of this activity pays off, supplies increase, and the price of oil begins to fall. The newer fleet of smaller, more fuel-efficient vehicles also starts to reduce demand, leading to further price

declines. This is a classic boom and bust cycle, in some ways analogous to the predator-prey model, but there is an additional wrinkle.

Consider the wildcatter or small independent operator that has bet a fortune on finding a new oil field or developing new techniques to increase the efficiency of oil extraction from existing fields or exploit new types of deposits.   These operators may have purchased a new drilling rig, or paid a large lease fee to rent one (many other people are trying to do the same thing at the same time, explaining the high rental cost), or invested heavily in research into new extraction techniques. These operators would also likely have had to pay large up-front fees to rent the land they are exploring or to obtain drilling permits. Just about the time the price of oil starts to fall, our small operator strikes oil. However, unlike classic supply-demand situations, there is now little incentive to turn off the taps and reduce supply – all the upfront costs associated with research, exploration, drilling, and bringing in a new well have already been paid. The marginal cost of keeping the well open and continuing to supply oil to the market is quite small. Shutting down the well might be an option for a major oil company, but not for a small operator - it would mean immediate bankruptcy. Instead, the operator is likely to hang on, hoping that the price of oil will rise. The net effect of this is that as the price of oil falls, supplies continue to increase, leading to even steeper price declines. People again start to buy gas-guzzlers, economists proclaim the wisdom of markets, promoters of fossil fuel suggest that we have infinite supplies, and the cycle starts again. Of course, as discussed in Chapters Six, Seven and Eight, there are environmental constraints to continued use of fossil fuels that are not reflected in simple models of supply and demand, predator and prey, or boom and bust.

6-6. The answer to this question is similar to the answer to question 6-5, namely that there is a lag between the time when price rises in the commodity markets signal the need for new supplies, and the time it takes to bring a new mine into production – this can easily be a decade or more. One additional factor in the mining industry compared to the oil industry is that because mines tend to be large and visible, and leave so many waste products at the surface, nearby inhabitants are more likely to oppose the mine because of the risk of environmental degradation or damage to water supplies. Local opposition can add many years to the time it takes to gain regulatory approval for a new mine. Many mineral commodities experience large price swings and "boom and bust" cycles.

7-1. For this problem, we will assume that the subsidence rate in 2005 was 20 mm/yr, and that the levee was constructed in 1965, 40 years earlier. We wish to find an acceptable model that satisfies the data. Other than time, there are two adjustable parameters in the model, z_0 and τ. There are infinite numbers of models that might work. However, the problem asks you to fix z_0, simplifying the problem. Hence, we will modify τ, the time constant, such that after 40 years, the subsidence rate equals 20 mm/yr.   If you are a student of calculus, this is a straightforward problem. Set the time derivative of height, $dz(t)/dt$, equal to 20 mm/yr, set time, t, equal to 40 years, and solve for τ. You should get $\tau=20$ years. Then using the original equation, calculate the height at $t=40$ years (2.1 meters below sea level). That number, added to the original height (0.5

meters above sea level) is the total amount of subsidence, 2.6 meters, experienced by the levee since it was built.

If you are not a student of calculus, don't despair, there is another way. Use a hand calculator and the equation in the figure caption to make a table with three columns. The first column is time (t), from zero to 40 years, using increments of 5 years (0, 5, 10, etc.). The second column is height, z, in meters, calculated from the equation. In the third column, you are going to approximate the rate of subsidence by simply taking the difference in height between each 5 year increment, and dividing by five years. We could get a more accurate answer if we calculated the height at every year, but the table would be five times as long, and five year increments are good enough. Make several tables by varying τ until you get a rate at t=40 years that is close to 20 mm/yr (hint-try values of τ shorter than 35 years). For a more accurate answer, try smaller time increments once you get close to 20 mm/yr.

With either approach, you should get at least 2 meters of subsidence, which would put the levees well below their design height. Levees that were too low undoubtedly contributed to the Katrina disaster.

7-2. This question asks you to first go to the open scientific literature to obtain key background information. Assuming that Tangier Island has been occupied since 1700, we wish to calculate the net amount of relative sea level rise due to the combination of GIA (land subsidence) and "absolute" sea level rise over the last 315 years (1700 to 2015). We can assume that the contribution from GIA-related subsidence has been constant. The data in *Karegar et al.* (2016) suggest that GIA contributes about 1.5 mm/yr of subsidence at this latitude (37° 50' North). 315 years time 1.5 mm/yr gives 47 cm, nearly half a meter of subsidence.

The amount of sea level rise over this period will be more difficult to calculate, as the rate has changed over this period. *Hay et al.* (2015) estimate 1.2 mm/yr for the most of the 20th Century. For this exercise, we will assume that this applies to a period shortly after the beginning of the industrial revolution, which we will arbitrarily take to be 1850. Prior to 1850, we will assume that sea level rise was zero. Figure 7.4 suggests that after 1990, the rate of sea level rise accelerated. Based on the data in this figure, we will assume that in the 25 years since 1990, the average rate of global sea level rise was 3.3 mm/yr. So for the three periods, we have:

Period	Amount of Sea Level Rise
1700 – 1850:	0
1850-1990: 1.2 mm/yr x 140 years =	168 mm
1990 – 2015: 3.3 mm/yr x 25 years =	82.5 mm
Total:	250.5 mm ~ 25 cm

Based on this simple exercise and our assumptions, in terms of understanding the problems of Tangier Island residents, GIA (47 cm) is almost twice as important as sea level rise (25 cm) associated with global warming.

Try the problem with different assumptions concerning the rate of sea level rise and timing, and see how it affects your answer. For example, what happens if you assume that the rate of sea level rise was 1.4 mm/yr beginning in 1775, the year the Watt-Bolton steam engine was commercialized?

7-3. First, this is rather a special case – Tangier Island happens to sit near the "bulls eye" for natural, GIA-induced subsidence. Many other coastal locations are also experiencing land loss and flooding, and sea level rise from global warming, rather than land subsidence, is the main cause. Second, the example above was chosen to illustrate what will happen to other coastal cities in the future, as sea level rise continues and accelerates. Third, while the flooding at Tangier Island may be largely due to GIA-related subsidence, global warming is still a factor. Moreover, while we can't do anything about GIA, we can do something about global warming.

8-1. The sun's maximum emittance occurs at about 0.48 μm, in the visible part of the spectrum. In contrast, the Earth radiates at a much longer wavelength, about 10 μm, in the infrared part of the spectrum (see Figure A 8.1).

8-2. Earth is 2.58 times farther from the Sun than Mercury (149.5 divided by 57.9). Hence it receives 6.67 times less solar radiation (2.58 squared), or about 1369 W/m², very close to the satellite-measured value.

8-3. Over 24 hours, the Earth rotates once, so all areas receive some solar radiation. In other words, the incoming solar flux gets distributed over the entire surface area of the Earth, $4\pi R^2$ (the surface area of a sphere) where R is Earth's radius, about 6,370 km. The total radiation that the Earth intercepts from the Sun is 1369 W/m² times the area of Earth's "disc" as seen from the sun, known as the effective cross section. This is just the area of a circle, πR^2 where R is again Earth's radius. The effective solar radiance is the total received (1369 W/m² times the effective cross section, πR^2) divided by the area over which it is distributed ($4\pi R^2$). The factor πR^2 appears in both the numerator and denominator and hence cancels, so the answer is just 1369 divided by 4, or 342 W/m².

8-4. We assume that the energy per unit time per square meter coming onto the Earth' surface (239 W/m²) is balanced by the amount of energy per unit time per square meter radiated from the Earth because of its temperature. Equation A8.3 (the Stefan-Boltzman law) tells us that this radiated energy only depends on temperature:

$$239 \text{ W/m}^2 = \sigma T^4$$
$$= 5.67 * 10^{-8} T^4$$

Re-arranging and solving for temperature gives 255°K, or -18°C (if your calculator does not do 4th roots, just take the square root twice). -18°C is pretty chilly – Earth's average temperature is closer to +15°C, due to the greenhouse effect, which we'll consider next.

8-5. For $\alpha = 0$, you should get 255°K (-18°C), the same answer as in Question 8-4. For $\alpha = 1$, the temperature is 303°K (+30°C), higher than Earth's average temperature, about 15°C. For $\alpha = 0.5$, T = 274°K (1°C). This suggests that the absorptivity of the atmosphere is somewhere between 0.5 and 1.0, probably close to 0.75. In other words, the atmosphere is about 75% efficient at trapping infrared radiation from Earth's surface. Adding CO_2 to the atmosphere increases that efficiency, and increases Earth's average surface temperature.

8-6. Using equation A 8.4, temperature would increase by about 3°C (the actual value is closer to 1°C).

8-7. The extra heat stored in the Earth's crust, ΔQ, is given by the mass of warmed rock (*M*), times the average heat capacity of that rock (*C*), times the temperature change (ΔT):

$$\Delta Q = MC\Delta T$$

To be accurate, we should integrate the anomalous temperature over the full depth range where the anomaly is observed, typically about 200 meters. For simplicity, we will just assume a constant anomalous temperature of 0.5 degrees C in the upper 100 meters. This is equivalent to assuming that the area of the triangular section of elevated temperature in Figure A 8.3 (base) can be approximated by an equivalent rectangular area. The volume of warmed rock is then 100 meters times the area of the continents, 2 $x10^{14}$ m^2, which is 2 x 10^{16} m^3. The mass is given by the volume times the average density. We'll assume a value of 2300 kg/m^3, somewhat less than fresh basalt or granite (2600-2900 kg/m^3) because the upper 100 meters is often fractured or altered, reducing average density. This gives a mass of warmed rock of 4.6 x 10^{19} kg. The heat capacity of rocks represents how much heat is required to raise the temperature of a given mass of rock by one degree). Although there are many different rock types sampled by bore holes and mines, it turns out that the heat capacity of most rocks is about the same: 10^3 J per kg per degree is a good average. Multiplying the mass of rock by this value and by the temperature change of 0.5° C gives $\Delta Q = $ 2.3 x 10^{22} J, about 3 times larger than the atmosphere. Changes in the atmosphere are just the tip of the proverbial iceberg when it comes to changes in Earth's overall heat balance.

8-8. These should be part of an in-class discussion. The referenced papers include useful material, but students should be encouraged to come up with their own ideas.

8-9. Most explanations for the correlation between CO_2 and temperature in Figure A 8.4 center on positive feedbacks, first discussed as part of the answer to question 8-6. These are mechanisms that promote natural release of CO_2 or other greenhouse gas such as methane into the atmosphere in response to small temperature changes. In other words,

small increases in solar insolation cause small increases in temperature, which stimulate natural release of CO_2, which stimulates more warming.

8-10. The feedback mechanisms listed above, such as tundra melting, can happen relatively quickly on geological time scales (tens to hundreds of years). In contrast, it takes thousands to tens of thousands of years for certain CO_2 removal mechanisms to operate. For example, weathering of continental rocks that release calcium can result in the sequestration of carbon in deep-sea sediments, as organisms form calcium carbonate shells, which fall to the sea floor, forming a rock called limestone. Therefore, CO_2 and temperature can increase relatively quickly, but both will tend to decrease slowly.

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